Remarkable Relations between Particle Masses

Fitting fermion masses into a Sudoku-like table using extended Koide formulae

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Abstract

In the context of the Standard Model of Particle Physics, the masses of elementary particles are believed to be risen by the Higgs mechanism, although the particle masses' exact values look arbitrary – they cannot be explained by any current theory, and it is still unclear if the different mass values are directly related at all.

In this paper, the author will demonstrate that it is possible to calculate relationships between the masses of all fermions, fitting them into a 12-cell table using the Koide formula[1] in a Sudoku-like manner with 6 different "target results" (3 for the different particle types, and 3 for the particle generations), those results being calculated using one simple formula and only 11 numbers (10 primes, plus the number 1):

$$\frac{m_1 + m_2 + m_3}{(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3})^2} = (\frac{p_1 \bullet p_2 \bullet p_3^x}{p_4})^y$$

The resulting table includes all fermion masses within the currently allowed ranges of experiment, while the 6 target results calculated using the Koide formula are matched with excellent precision (10^{-13} or better for 5 target results, and 10^{-6} for the remaining 6th target result).

The author is aware of the fact that those results could be purely coincidental, of course – the challenge of this work was to demonstrate that it is *technically possible* to build such a model, which does *not* necessarily mean that it should represent any *physical reality*.

Introduction

In 1981, Yoshio Koide discovered a formula relating the masses of the three charged leptons (with m_e = 0.510998946(3) MeV/c², m_μ = 105.6583745(24) MeV/c², m_τ = 1776.86(12) MeV/c²):

$$Q = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} \approx 0.666661 \approx \frac{2}{3}$$

Koide's result could be purely coincidental, of course, but it also looks quite remarkable as the resulting value is almost exactly a simple fraction, at the same time 2/3 lies exactly halfway between the two extremes allowed by the formula: 1/3 (should the three masses be equal) and 1 (should one mass dominate).[2]

In this paper, the author will show that it is possible to extend the use of the Koide formula so that remarkable results can also be found for quarks as well as for all three fermion generations (or "families").

Procedure

First, all fermions were placed in a table featuring dedicated rows for charged leptons, up-type quarks, down-type quarks, and neutrinos; as well as one column for each fermion generation, as seen in Table 1:

	Generation I	**Generation II**	**Generation III**	
charged leptons	electron	muon	tau	$\frac{2}{3} = 0.666\ldots$
up-type quarks	up quark	charm quark	top quark	$\frac{?}{?}$
down-type quarks	down quark	strange quark	bottom quark	$\frac{?}{?}$
neutrinos	electron neutrino	muon neutrino	tau neutrino	
	$\frac{?}{?}$	$\frac{?}{?}$	$\frac{?}{?}$	

Table 1: While the Koide result was known for charged leptons, the challenge was to find equally simple fractions for quarks (horizontally) and particle generations (vertically).

While the charged leptons' result was known (2/3 via the Koide formula), the challenge was now to find valid fractions for up-type quarks and down-type quarks, as well as for all three particle generations, using Koide-like formulae in all cases, which would allow to fit all particle masses in a Sudoku-like table.

It was clear that it would be impossible to get 2/3 as a result in the missing 5 cases, and so the goal was to find alternative fractions for the quarks and generations that would be as simple as possible, using only a minimum of small (prime) numbers.

Note that neutrinos masses were not considered when looking for fitting formulae, as they are supposed to be very small and still not known to a degree that would allow to calculate meaningful results.

In order to find fractions offering good matches for the experimental results, default results were calculated using experimental mass values[3] with the Koide formula, as shown in Table 2:

	Generation I	Generation II	Generation III	
charged leptons	electron 0.51099895000(15)	muon 105.6583755(23)	tau 1776.86(±0.12)	0.666660512412452
up-type quarks	up quark 2.2(+0.5/−0.4)	charm quark 1275(+25/−35)	top quark 173100(±400)	0.848842243879846
down-type quarks	down quark 4.7(+0.5/−0.3)	strange quark 95(+9/−3)	bottom quark 4180(+40/−30)	0.729999482914477
	0.3887794782322	0.475074336396146	0.654971530480622	

Table 2: Using the Koide formula, default values were calculated for each particle type and for each particle generation (neutrinos excluded).

A small computer program was developed, allowing to find promising matches using simple fractions, and the results of this search are represented in Table 3:

	Koide results, using currently available experimental data.	The fractions used in the model described in this paper.	Decimal values, calculated using the model's fractions.
charged leptons	0.666660512412452	$\frac{2}{3}$	0.666666666666667
up-type quarks	0.848842243879846	$\frac{2 \bullet 59}{139}$	0.848920863309352
down-type quarks	0.729999482914477	$\frac{3^3}{37}$	0.72972972972973
generation 1	0.3887794782322	$\frac{7}{2 \bullet 3^2}$	0.388888888888889
generation 2	0.475074336396146	$\frac{1709}{2 \bullet 7 \bullet 257}$	0.474986103390773
generation 3	0.654971530480622	$\frac{19}{29}$	0.655172413793103

Table 3: Using the simple Koide results calculated using experimental data, the author started to search for simple fractions that would produce numbers that would come close to the calculated results.

The model requires a total of 10 prime numbers to calculate the 6 target values for all particle classes and generations, as seen in Table 4:

Variable	Value	Prime	Instances		Variable	Value	Prime	Instances
a	2	*1nd*	5		**f**	37	*12th*	1
b	3	*2nd*	4		**g**	59	*17th*	1
c	7	*4th*	2		**h**	139	*34th*	1
d	19	*8th*	1		**i**	257	*55th*	1
e	29	*10th*	1		**j**	1709	*267th*	1

Table 4: The 10 prime numbers used in the model described in this paper.

The fitting fractions looked arbitrary at first, but as soon as prime numbers will be replaced by variables (*a–j*, as seen in Table 4) and the fractions will be re-written, it becomes clear that the fractions are not only quite similar, but that it is even possible to see a basic symmetry between the fractions used for the particle types and the particle generations, as seen in Tables 5 and 6.

	The fractions used to calculate the target values for the Koide formulae.	**The same fractions, values replaced by variables.**	**Re-writing the fractions visualizes similarities and symmetries.**
leptons	$\frac{2}{3}$	$\frac{a}{b}$	$\frac{a}{b}$
up-type quarks	$\frac{2 \bullet 59}{139}$	$\frac{a \bullet g}{h}$	$\frac{1 \bullet a \bullet g}{h}$
down-type quarks	$\frac{3^3}{37}$	$\frac{b^3}{f}$	$\frac{b \bullet b^2}{f}$
generation 1	$\frac{7}{2 \bullet 3^2}$	$\frac{c}{a \bullet b^2}$	$(\frac{a \bullet b^2}{c})^{-1}$
generation 2	$\frac{1709}{2 \bullet 7 \bullet 257}$	$\frac{j}{a \bullet c \bullet i}$	$(\frac{a \bullet c \bullet i}{j})^{-1}$
generation 3	$\frac{19}{29}$	$\frac{d}{e}$	$(\frac{e}{d})^{-1}$

Table 5: Replacing the values by variables (column 3) and finally re-writing the fractions (column 4) makes it easier to understand their relations.

It is interesting to see that the fractions used to calculate the generation values seem to inverted versions of the fractions used to calculate the particle type values:

	horizontal	**vertical**	**number of required values**
charged leptons, as well as generation 3	$\frac{a}{b}$	$(\frac{e}{d})^{-1}$	2
up-type quarks, as well as generation 2	$\frac{1 \bullet a \bullet g}{h}$	$(\frac{a \bullet c \bullet i}{j})^{-1}$	4
down-type quarks, as well as generation 1	$\frac{b \bullet b^2}{f}$	$(\frac{a \bullet b^2}{c})^{-1}$	3

Table 6: In the end, we only have 3 different types of formulae, as there is a symmetry between particle type and generation formulae.

Finally, it will be possible to write down a single formula that can be used to calculate the target values for all particle types and particle generations:

$$\frac{m_1 + m_2 + m_3}{(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3})^2} = (\frac{p_1 \bullet p_2 \bullet p_3^x}{p_4})^y$$

The possible values for p_1, p_2, p_3, p_4, x and y can be seen in Table 7:

	p_1	**p_2**	**p_3**	**p_4**	**x**	**y**
charged leptons	1	1	2	3	1	1
up-type quarks	1	2	59	139	1	1
down-type quarks	1	1	3	37	3	1
generation 1	1	2	3	7	2	-1
generation 2	2	7	257	1709	1	-1
generation 3	1	1	29	19	1	-1

Table 7: 6 • 6 = 36 values can be inserted into the formula, defining possible values for the particle masses via the Koide formula.

The values calculated using the formula noted above have been inserted into table featuring all fermions, and a small computer program was created that allowed to manually adjust and fine-tune particle masses, as seen in Table 8:

	Generation I	**Generation II**	**Generation III**	
charged leptons	electron	muon	tau	$\frac{2}{3}$
mass (model) mass (experiment)	0.51099895000 0.51099895000(15)	105.6583755 105.6583755(23)	1776.96902708302 1776.86(±0.12)	0.666666666666667 0.666666666666667
up-type quarks	up quark	charm quark	top quark	$\frac{2 \bullet 59}{139}$
mass (model) mass (experiment)	2.1929193980218 2.2(+0.5/−0.4)	1274.27601 1275(+25/−35)	173196.251279621 173100(±400)	0.848920863309352 0.848920863309352
down-type quarks	down quark	strange quark	bottom quark	$\frac{3^3}{37}$
mass (model) mass (experiment)	4.7049105641197 4.7(+0.5/−0.3)	95.05060627355 95(+9/−3)	4171.00518631075 4180(+40/−30)	0.72972972972973 0.72972972972973
neutrinos	electron neutrino	muon neutrino	tau neutrino	
	0.000… (?)	0.000… (?)	0.000… (?)	?
	$\frac{7}{2 \bullet 3^2}$	$\frac{1709}{2 \bullet 7 \bullet 257}$	$\frac{19}{29}$	
mass (model) mass (experiment)	0.388888888888889 0.388888888888889	0.474986065351445 0.474986103390773	0.655172413793103 0.655172413793103	

Table 8: The completed table, which shows the differences between the model's calculated particle masses, compared to the masses determined by experiment.

Masses of up-type quarks and down-type quarks have been accumulated using the model's formula (with includes the original Koide formula), while the masses of each particle generation has been accumulated using an extended version that allows to include neutrino masses:

$$\frac{m_e + m_u + m_d + m_n}{(\sqrt{m_e} + \sqrt{m_u} + \sqrt{m_d} + \sqrt{m_n})^2} = (\frac{p_1 \bullet p_2 \bullet p_3^x}{p_4})^y$$

Note that neutrino masses are so small compared to other masses that this model will not allow to make any reasonable predictions.

Also note that the computer program did not allow to automatically calculate allowed value ranges, all possible combinations had to be manually tested. Writing a more advanced program that would allow to automatically calculate ranges could be done in a following step.

The target values used in the model are:

	Fraction / target	Value (fraction result)	Value (manually fine-tuned masses)	Precision
leptons	$\frac{2}{3}$	0.666666666666667	0.666666666666667	10^{-13}
up-type quarks	$\frac{2 \bullet 59}{139}$	0.848920863309352	0.848920863309352	10^{-13}
down-type quarks	$\frac{3^3}{37}$	0.72972972972973	0.72972972972973	10^{-13}
generation 1	$\frac{7}{2 \bullet 3^2}$	0.388888888888889	0.388888888888889	10^{-13}
generation 2	$\frac{1709}{2 \bullet 7 \bullet 257}$	0.474986103390773	0.474986065351445	10^{-6}
generation 3	$\frac{19}{29}$	0.655172413793103	0.655172413793103	10^{-13}

Table 9: The model described in this paper can be used to fine-tune particle masses in a way that the Koide formula results are very close to the calculated target values (precision of 10^{-6} to 10^{-13}).

Particle masses allowed by the model described in this paper are very close to the masses detected in experiments, and they are all in the ranges of possible experimental errors, as shown in table 10:

Generation	Particle	Experiment	Value allowed by the model described in this paper
I	electron	0.51099895000(15)	0.51099895000
	up quark	2.2(+0.5/−0.4)	2.1929193980218
	down quark	4.7(+0.5/−0.3)	4.7049105641197
II	muon	105.6583755(23)	105.6583755
	charm quark	1275(+25/−35)	1274.27601
	strange quark	95(+9/−3)	95.05060627355
III	tau	1776.86(±0.12)	1776.96902708302
	top quark	173100(±400)	173196.251279621
	bottom quark	4180(+40/−30)	4171.00518631075

Table 10: Comparing the values inserted in the model described in this paper with recent experimental values.

Notes by the author

1. The author of this paper is not a scientist, and the author would like to apologize in advance for all kinds of formal mistakes he will probably have made when writing this paper.

2. The author is aware of the fact that the results shown in this paper could be purely coincidental, and the author does not claim that the results represent any physical reality. The sole goal of writing this paper was to show that particle masses could technically fit into a table as described in this paper.

3. The author is aware of the fact that current fermion mass measurements may not be perfect and that different experiments are offering slightly different results that may also slightly differ from the exact numbers used in this paper.

4. The author is aware that others (F. G. Cao, Rodejohann, Zhang) also have found remarkable mass relationships using the Koide formula. The heaviest three quarks (charm, bottom, top) give the value 0.669, which is roughly 2/3. The lightest quarks (up, down,strange) give the value 0.56, which is roughly 5/9.

5. While working on this paper, the author has found similar relationships between W/Z and Higgs bosons. Inserting the W, Z and Higgs boson masses into the Koide formula gives a result of 0.33633251332269, which is roughly 1/3. Inserting the W boson's mass twice (considering both W^+ and W^-) gives a value of 0.252207181698337, which is roughly 1/4.

6. The original Koide formula produces a result of 0.666661(7), which is very close to 2/3 (it is off by a factor of only 1.0000085). In a similar way, the fine structure constant is 0.0072973525693(11), which is very close to 1/137 (off by a factor of only 1.000262767). The author's personal opinion is that it's quite remarkable that some of nature's most basic values are so close to simple fractions involving prime numbers, and that this matter should be further investigated. It would be interesting to see if those results are really purely coincidental – and if they are not, then it would be imperative to find out why the results are that close, but not equal to fractions involving prime numbers. If those results are not coincidental, then they could definitely point out to new physics beyond the Standard Model.

7. The major weakness of the model presented in this paper is that while the author could show that the particle mass values could fit, they still cannot be calculated. It would be interesting to write a computer program that would calculate the allowed particle mass ranges using the formulae and prime numbers presented in this paper.

8. This paper has shown that particle mass values that look completely arbitrary and unrelated will fit into a Sudoku-like table using only one quite simple formula fed with only 11 numbers (number 1, plus 10 prime numbers). Nevertheless, both the selection of prime numbers as well as their positions within the formula still look arbitrary, and the author was not able to find meaningful relationships between the prime numbers used (2, 3, 7, 19, 29, 37, 59, 139, 257, and 1709). Searches on the On-Line Encyclopedia Of Integer Sequences® (oeis.org) did not return any valuable results.[4]

9. The author thinks that it may be interesting to see if it could be possible to set up a similar table using even smaller prime numbers. Especially 1709 (which is the 267th prime) doesn't seem to fit well into the sequence.

10. Future experiments will show if more precise measurements of particle masses will still fit into the table described by the model in this paper, or not.

References

[1] Basic information on the original Koide formula:

The strange formula of Dr. Koide
Alejandro Rivero and Andre Gsponer, 2008
https://arxiv.org/abs/hep-ph/0505220

[2] Further information on the original Koide formula*:

https://en.wikipedia.org/wiki/Koide_formula

[3] Particle masses can be found online. It should be noted that the values may slightly differ, depending on the sources and the experiments. The values used in this paper can be found here, for example*:

https://en.wikipedia.org/wiki/List_of_particles

[4] The author has been using the On-Line Encyclopedia Of Integer Sequences® to lookup matching lists of prime numbers:

https://oeis.org

*) The author is aware of the fact that it should be avoided to cite Wikipedia in scientific papers, although the information on this page should be trustable.